Online supplement for 10.1176/appi.ps.202100587

1. Detailed Analyses

The composition of enrollment for Black individuals is the percentage of clients who enroll in a particular site whose race is Black, calculated as

$$Composition_{black\ at\ a\ particular\ site} = \frac{Number\ of\ black\ participants\ at\ site}{Total\ number\ of\ participants\ at\ site} \times 100 \tag{1}$$

Similarly, the composition for the Black group in the site's service area, $Composition_{black\ at\ the\ service\ area\ of\ this\ site}$, is also calculated. Instead of comparing the difference between the two compositions , the relative difference in composition is used to show the degree of the difference with respect to the composition at the service area. This relative difference[1] is calculated as

$$Relative\ difference = \frac{Composition_{black\ at\ a\ particular\ site} - Composition_{black\ at\ the\ service\ area\ of\ this\ site}}{Composition_{black\ at\ the\ service\ area\ of\ this\ site}} \tag{2}$$

If the relative difference in composition is 0 or close to 0, that means there is no or low disproportionality. If the relative difference equals 1, that means the composition of Black clients at the site is twice as much as would be expected from the composition of Black population at the service area, if it equals 2, then it is three times as much, and so on.

The risk of site enrollment for the Black population is expressed as a percentage of the Black population in the service area who participate at the site. It is calculated as

$$Risk_{black} = \frac{Number\ of\ black\ participants\ at\ a\ particular\ site}{Total\ number\ of\ black\ population\ at\ the\ service\ area\ of\ this\ site} \times 100 \tag{3}$$

[1] The relative difference is expressed as a proportion instead of percentage as defined in the IDEA guide.

Similarly, the risk for the non-Black population is also calculated using equation (3). The risk ratio is then calculated to describe how the risk for the Black population compares to the risk for the non-Black population.

$$Risk\ ratio = \frac{Risk_{black}}{Risk_{non-black}} \tag{4}$$

A risk ratio that is equal to or is close to 1 indicates that the risk between the Black and non-Black population is close or equal, that is, no or low disproportionality. A risk ratio of 0 occurs if there are no Black clients in a site and it indicates high disproportionality of the non-Black population. When all clients are Black, the risk ratio cannot be calculated because the denominator $Risk_{non-black}$ is equal to 0.

2. Methodology for Generating 30 Replicated Tables

A goal of the replicated tables approach for the application is to arrive at a set of 30 tables[2] such that the variation across the tables is approximately the same as the estimated variance associated in a single published table. The Dirichlet distribution is useful for simulating contingency tables since it takes into account the relationship between the table cells. The static table has 35 interior table cells (total estimated number of Blacks in the service area for each of the sites), with weighted cell frequency X_k, and associated variance. The cell proportion in cell k can be expressed as $p_k = \frac{X_k}{X}$, where we assume the overall weighted frequency X for the table follows a normal distribution, $N(\mu, \sigma^2)$. Assume that the cell proportions $(p_1, p_2, \dots p_k, \dots, p_{35})$ are the realizations of a set of random variables that follow a Dirichlet distribution model with parameters $(\alpha_1, \alpha_2, \dots \alpha_k, \dots, \alpha_{35})$. The means and variances of the cell proportions are $E(p_k) = \frac{\alpha_k}{\alpha_0}$ and $Var(p_k) = \frac{\alpha_k(\alpha_0 - \alpha_k)}{\alpha_0{}^2(\alpha_0+1)}$, where $\alpha_0 = \alpha_1 + \alpha_2 + \cdots + \alpha_K$. The steps to generate the replicated tables are as follows:

1. The overall weighted frequency, X, is drawn from a normal distribution.

[2] The choice of generating 30 tables is supported in general by the Central Limit Theorem, which approximates a normal distribution among the estimates as the sample size becomes larger, assuming that all samples are identical in size, and regardless of the population distribution shape. A sample size of at least 30 is widely accepted to satisfy the Central Limit Theorem.

2. The set of cell proportions, p_k's , is drawn from the Dirichlet distribution.
3. The cell counts of a replicated table are derived as $X_k = Xp_k$.
4. Steps 1 to 3 are repeated 30 times to generate 30 replicated tables.

After producing the 30 replicated tables, the resulting variation among the replicated tables is checked.

The parameters of the normal distribution and the Dirichlet distribution need to be estimated to generate the replicated tables. A distance function method is used to estimate the parameters $(\alpha_1, \alpha_2, \dots \alpha_k, \dots, \alpha_{35})$ of the Dirichlet distribution. The estimation of the parameters is facilitated through a generalized variance function (GVF) method. GVFs have been used in large national surveys mainly as a way to easily estimate the variance associated with a resulting point estimate. The GVF is a curve of the form $V_{X_k}^2 = a + \frac{b}{X_k}$, where $V_{X_k}^2 = \frac{var(X_k)}{X_k^2}$ are estimated relative variances, and *a* and *b* are parameters to be estimated by an least squares regression process.

The variances (and relative variances) are not available for the 35 service area proportions, so a GVF was fitted to available estimates from the American Community Survey with similar sizes as the Black population in the service areas, which included estimates from all counties in the U.S. Because the GVF as given above, which is a linear regression, is susceptible to generate negative variances, a no-intercept model was fit of the form $V_{X_k}^2 = \frac{b}{X_k}$, where b = 105.56. The model parameter *b* was applied to the data from the 35 sites to arrive at the estimated variances.

For a proportion, $p_k = \frac{X_k}{X}$, where *X* represents an estimate for a certain subpopulation, the estimated relative variance is approximated as $V_{p_k}^2 \approx V_{X_k}^2 - V_X^2$. This leads to $var(p_k) = p_k^2(\frac{var(X_k)}{X_k^2} - \frac{var(X)}{X^2})$. A special case results when using the same GVF function for both X_k and *X*, which results in $var(p_k) = \frac{b}{X} p_k(1 - p_k)$. Note that the use of this formula assumes independence between the numerator X_k and the denominator *X*. The parameters of the Dirichlet distribution can be estimated from the observed cell proportions $(p_1, p_2, \dots p_k, \dots, p_{35})$. Set $\hat{\alpha}_k = \left(\frac{X}{b} - 1\right) p_k$. As a result, $var(p_k) = \frac{\hat{\alpha}_k(\hat{\alpha}_0 - \hat{\alpha}_k)}{\hat{\alpha}_0{}^2(\hat{\alpha}_0 + 1)} = \frac{b}{X} p_k(1 - p_k)$, where $\hat{\alpha}_0 = \hat{\alpha}_1 + \cdots + \hat{\alpha}_{35}$.

The parameters of the normal distribution μ and σ^2 can be estimated by the overall weighted total and its associated variance. The overall variance is estimated from the same GVF formula described above. More details about the replicated tables methodology can be found in Krenzke and Li (2019).

To provide a sense of the uncertainty around the estimated service area percentages, error bounds were computed for each site. The error bounds represent the 10th and 90th percentiles among the set of 30 estimates generated from the replicated tables methodology. Also, for each of the four scenarios (unadjusted status, if next client is Black, if next five clients are Black, and if the next client is non-Black), the number of times across the 30 replicated tables is tallied with regards to being among top 5 and top 10.

3. Results of Sensitivity Analysis

As mentioned above, sensitivity analyses can address the issues of sampling error in the estimated service area population percentage who are Black and the low number of clients in some sites. To address the first issue, the error bounds that appear in Figure 3 represent the 10th and 90th percentiles of the replicated estimates for the two analysis metrics. Error bounds that do not contain the estimates of reference (0 for the estimates of relative difference in composition and 1 for the estimates of risk ratio) indicate potential disproportionality. The results with the error bounds confirm the findings of disproportionality reported above.

To address the second issue, we recalculated the 30 sets of replicated estimates to show the number of times that the replicated estimates is among the top 5 and top 10 of all 35 sites, both for the unadjusted status and under the hypothesized scenarios developed to account for small site sizes (see Tables 2 and 3). For the relative difference in composition estimates under the unadjusted status scenario, seven sites (Site IDs 14, 18, 21, 22, 23, 28, and 32) always show up in the top 10, and three sites (Site ID 8, 11, and 29) are in the top 10 a majority of the time. If the next client coming into the site is non-Black, the results are similar to the unadjusted status. However, if the next client(s) coming into the site is (are) Black, the sites with top estimates change and new sites (Site IDs 6, 24, 26, 27, 31, 33, and 35) appear in the list. These are the sites originally with zero Black clients, indicating that the unadjusted disproportionality estimates (i.e., -1 relative difference in composition, and 0 risk ratio) of sites with no Black clients are very sensitive to incoming clients. Therefore, the disproportionality estimates for these particular seven sites tend to be unstable and should be considered with caution.

The estimates of risk ratios are similar to that of the relative difference in composition. Sites 28 and 32 have the highest risk ratios. If the next incoming client(s) is (are) Black, Sites 27 and 33 become the top sites with highest risk ratios.